

# B meson decay constant with the Wilson and Clover heavy quark actions\*

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We present results of our quenched study of the  $B$  meson decay constant obtained with a parallel set of simulations with the Wilson and Clover actions at  $\beta=5.9, 6.1$  and  $6.3$ . Systematic errors associated with the large  $b$ -quark mass are analyzed within the Fermilab non-relativistic formalism. As our best estimate in the continuum limit we obtain  $f_B=163\pm 16$  MeV and  $f_{B_s}=175\pm 18$  MeV with the Clover action.

## 1. Introduction

A reliable determination of the  $B$  meson decay constant is a subject yet to be completed in lattice QCD. We have been pursuing this goal employing the relativistic formalism for heavy quark. Our results for the Wilson action has been reported in Refs. [1,2]. Since Lattice'96 we have carried out a parallel set of simulations with the  $O(a)$ -improved Clover action. We have analyzed the results for the two actions within the Fermilab non-relativistic formalism[3] with the view to understand the systematic error due to a large value of  $b$ -quark mass. In this article we report a summary of results from the simulations and analyses.

## 2. Simulation

The parameters of our simulations are listed in Table 1. For the clover coefficient we use the tadpole-modified one-loop value[5] given by  $c_{sw}=P^{-3/4}[1+0.199\alpha_V(1/a)]$  with  $P$  the average plaquette. Heavy quarks are simulated for 7 values of the hopping parameter  $\kappa$  covering the  $c$  and

Table 1

Simulation parameters. The lattice scale quoted is fixed by the string tension  $\sqrt{\sigma}=427$  MeV[4].

action	$\beta$	5.9	6.1	6.3
size		$16^3\times 40$	$24^3\times 64$	$32^3\times 80$
$1/a$ (GeV)		1.60(1)	2.29(1)	3.05(2)
$L$ (fm)		2.0	2.1	2.1
Wilson	$N_{conf}$	150	100	100
Clover	$N_{conf}$	540	200	166
	$c_{sw}$	1.580	1.525	1.484

$b$  quark masses, and 4 values of  $\kappa$  are employed for light quark.

The heavy-light decay constant  $f_P$  is extracted from the correlators of the axial vector current  $A_4$  and a smeared pseudoscalar density  $P^S(x) = \sum_{\vec{r}} \phi(|\vec{r}|) \bar{Q}(x+\vec{r}) \gamma_5 q(x)$  on the Coulomb gauge fixed gluon configurations, where  $\phi(|\vec{r}|)$  is the pseudoscalar wave function measured for each heavy and light quark masses.

A new perturbative ingredient in our work is the recent one-loop result[6] for the pole ( $m_1^Q$ ) and kinetic ( $m_2^Q$ ) masses of heavy quark and the renormalization factor  $Z_A(am_Q)$  of the axial vector current for finite bare heavy quark mass  $m_Q$

\*presented by S. Hashimoto

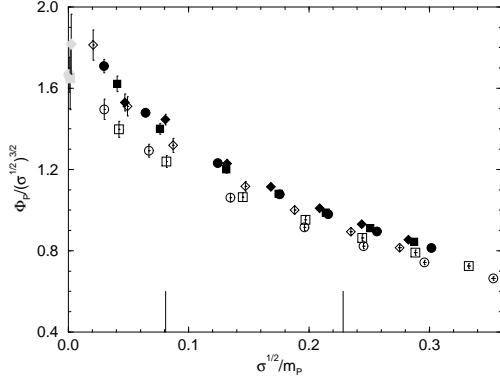


Figure 1.  $\Phi_P / \sqrt{\sigma}^{3/2}$  as a function of  $\sqrt{\sigma} / m_P$ . Filled (open) symbols are Clover (Wilson) results. Circles, squares and diamonds correspond to values at  $\beta=5.9, 6.1$  and  $6.3$ .

(see Refs. [7,8] for previous calculation for the Wilson case). Effects of finite values of  $am_Q$  in  $Z_A$  is significant, reducing  $f_B$  by 5–2% for the Wilson action and increasing it by a similar magnitude for the Clover case compared to the value obtained with  $Z_A(am_Q = 0)$ .

We define the heavy-light meson mass by[9,2]  $m_P = m_{P1} + m_2^Q - m_1^Q$  with  $m_{P1}$  the measured meson pole mass and the one-loop perturbative result[6] applied for  $m_2^Q - m_1^Q$ . This definition does not have the problem of the measured kinetic mass that the  $b$  quark mass can not be determined consistently from heavy-light and heavy-heavy mesons [10,11,2].

### 3. Results

We plot the quantity defined by  $\Phi(m_P) = (\alpha_s(m_P) / \alpha_s(m_B))^{2/\beta_0} f_P \sqrt{m_P}$  in Fig. 1 as a function of  $1/m_P$  for the the Clover(filled symbols) and Wilson (open symbols) actions. The light quark mass is linearly extrapolated to the chiral limit, and  $\alpha_s(\mu)$  is calculated with the standard 2-loop definition with  $\Lambda_{QCD} = 295$  MeV. We normalize the results by the string tension  $\sigma$ [4] since we primarily wish to examine the question of large- $am_Q$  errors in this figure. Vertical lines indicate the position of  $B$  and  $D$  mesons for  $\sqrt{\sigma} = 427$  MeV. Data points at  $1/m_P = 0$  are the static results[12], to which our results for the same set of  $\beta$  values converge.

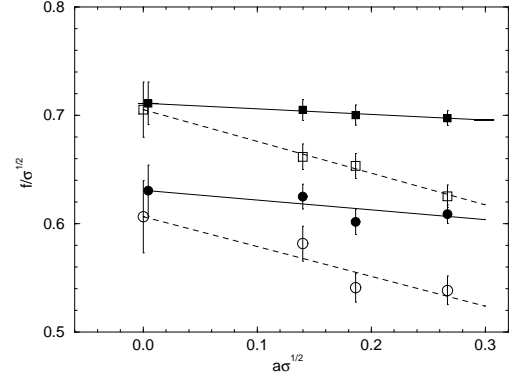


Figure 2. Continuum extrapolation of  $f_B$  (circles) and  $f_D$  (squares). Filled (open) symbols are Clover (Wilson) results.

In Fig. 2 we plot the continuum extrapolation of  $f_B$  and  $f_D$ . The Wilson results exhibit a scaling violation of 11–5% in our range of lattice spacing  $a^{-1} \approx 1.6$ –3 GeV, while the Clover results show a significantly reduced variation of 4–2%. These magnitudes are common to  $f_B$  and  $f_D$ . Furthermore the continuum values obtained with the two actions by a linear extrapolation agree within the statistical error of about 5%.

We emphasize that this agreement does not necessarily mean that systematic errors due to heavy quark are negligibly small. In the non-relativistic interpretation, the equivalent Hamiltonian for Wilson-type actions has the form

$$H = \bar{Q} \left[ m_1 - \frac{\vec{D}^2}{2m_2} - \frac{i\vec{\sigma} \cdot \vec{B}}{2m_B} + O(1/m_Q^2) \right] Q. \quad (1)$$

For the Wilson action for which  $m_B \neq m_2$ , the leading error in the decay constant due to heavy quark is  $O((c_B - 1)\Lambda_{QCD}/m_Q)$  with  $c_B = m_2/m_B$ . For the  $B$  meson an examination  $c_B$  at the tree level shows that a linear extrapolation of  $c_B$  from our range of  $a^{-1}$  leads to a value  $|c_B - 1| \approx 0.4$  at  $a=0$ . We should therefore allow an  $O(3\%)$  error unremoved in the continuum limit where we used  $\Lambda_{QCD}=0.3$  GeV. The same magnitude of error also remains for  $f_D$ .

There are two more sources of systematic error we need to consider. One is  $m_Q$ -independent scaling violation of  $O(a\Lambda_{QCD})$ , which we estimate to be 10% at our smallest  $a^{-1} \approx 3$  GeV. The other is  $O(\alpha_s^2)$  uncertainty due to the use of one-loop

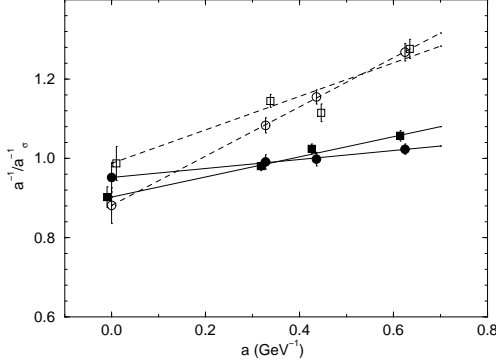


Figure 3. Ratio of lattice scale obtained from  $m_\rho$  (circles) and from  $f_\pi$  (squares) to that from the string tension. Filled (open) symbols are for Clover (Wilson) action.

value for  $Z_A$ , which is  $O(4\%)$  with  $\alpha_s \approx 0.2$ . Adding all the errors by quadrature leads to a combined systematic error of  $O(11\%)$  in the decay constant for the Wilson case.

For the Clover action for which  $m_B = m_2$  to  $O(\alpha_s)$ , the large- $am_Q$  errors have the form  $O(\alpha_s \Lambda_{QCD}/m_Q)$  and  $O(\Lambda_{QCD}^2/m_Q^2)$ . We estimate their magnitude to be  $O(1\%)$  incorporating the effect of coefficients, similar to  $c_B$ , that vanish in the continuum limit. The scaling violation errors are  $O(\alpha_s a \Lambda_{QCD})$ ,  $a^2 \Lambda_{QCD}^2$  which are small at  $O(2\%)$ . With the 2-loop error of  $O(4\%)$  from  $Z_A$  and an additional error of  $O(2\%)$  arising from the field rotation ignored in the present calculation, the combined systematic error amounts to  $O(5\%)$  for the Clover case.

So far we have used the string tension  $\sigma$  to set the scale. In Fig. 3 we plot the ratio of  $a^{-1}$  obtained with  $m_\rho$  and  $f_\pi$  to that with  $\sigma$ . We use the variation of the ratio to estimate the uncertainty in setting the scale, which we take to be 10% for the Wilson case and 5% for the Clover case. This uncertainty includes possible quenching error as the ratio need not converge to unity in the continuum limit.

Our final result for the decay constant is tabulated in Table 2. To obtain the values we take the continuum extrapolation of  $f_P/\sqrt{\sigma}$  and convert it  $f_P/m_\rho$  with the value of  $\sqrt{\sigma}/m_\rho$  at  $a=0$  in Fig. 3. A direct continuum extrapolation of  $f_P/m_\rho$  yields consistent results. The quoted errors are statistical, systematic and due to scale

Table 2

Results for the decay constant in MeV unit.

	Wilson	Clover
$f_B$	140(11)(15)(24)	163(9)(8)(11)
$f_{B_s}$	159(10)(17)(27)	175(9)(9)(13)
$f_D$	163(13)(18)(28)	184(9)(9)(12)
$f_{D_s}$	180(11)(20)(31)	203(9)(10)(14)

setting as estimated above in this order. We take the result for the Clover action to be our best estimate primarily because scaling violation is smaller and also since the statistical ensemble is larger compared to the Wilson action. Combining errors by quadrature we obtain the results quoted in the abstract.

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